NUMERICAL STUDY OF THE EFFECT OF FREE CONVECTION ON THE DEVELOPMENT OF VERTICAL TURBULENCE IN A SEMIINFINITE GAS JET

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A study is made of the role of buoyancy in turbulent flow in a semiinfinite gas jet. The development of the characteristics of the mean and pulsating flows is analyzed.

The flow induced by the action of buoyancy may significantly affect the dynamic characteristics and heat exchange in forced convection in a number of cases. In engineering practice it is often necessary to study mixed convection in the case of stream flow over solid surfaces. One feature of such flows, together with decaying forced convection, is the development of free-convective flow due to heating of the wall.

Until recently, most investigators used the mixing length to predict the structure of flows in stratified media [1, 2]. In [3], models of a higher rank are presented. These models are based on differential equations for the quantities characterizing the pulsative flow field: turbulent kinetic energy (TKE), $K = \langle u_i' u_i' \rangle/2$, the rate of "isotropic" dissipation RID, $\varepsilon = 2\nu l \langle \frac{\partial u_i'}{\partial x_j} \frac{\partial u_i'}{\partial x_j} \rangle$, and the mean square of the temperature pulsation $\langle T'^2 \rangle$. The authors of [4, 5] developed an algebraic approach to modeling the Reynolds stresses $\langle u_i' u_j' \rangle$ and the turbulent heat flux $\langle u_i' T' \rangle$ (AMS). Using a similar model as a basis, Chen and others calculated the characteristics of a free vertical jet in the near-range [6] and long-range [7] field of action of buoyancy. The results were correlated so as to follow the development of free convection in zones of jet flow without the influence of volume changes, mixed convection, and the predominance of free-convective flow. The influence of buoyancy effects in circular free jets was studied in [8, 9]. We should note the work [10] for boundary flows, where the AMS method was used to study free-convective flows from heated walls.

The goal of the present work is to study the flow structure and heat exchange in a vertical turbulent semiinfinite gas jet in a gravity field. The works [11, 12] examined the properties of the solution of a similar problem for laminar flow with a different orientation of the submerged plate.

Figure 1 shows a diagram of the two-dimensional flow being studied. A plane gas jet is discharged with a certain initial velocity u_0 from a nozzle of finite dimension L_0. The flow develops along a vertical wall heated to the temperature T_w. The temperature of the still surrounding medium is equal to the temperature of the jet at the nozzle outlet. We will analyze steady flow, satisfying boundary-layer approximations. In this case, the problem reduces to integration of the following system of equations:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0, \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu_{ef} \frac{\partial u}{\partial y}\right) + g(\rho_\infty - \rho), \tag{2}$$

$$\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\mu_{ef}}{\sigma} \frac{\partial T}{\partial y}\right). \tag{3}$$

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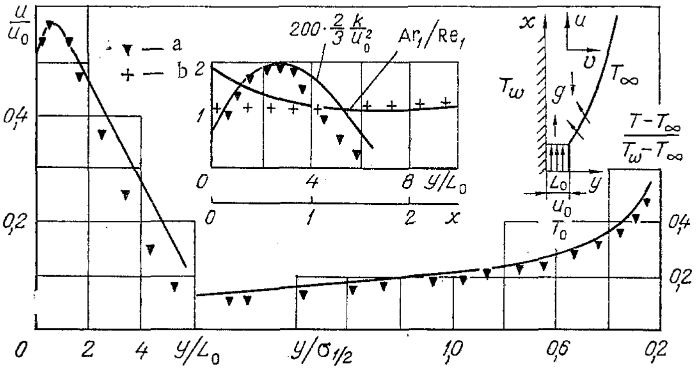

Fig. 1. Comparison of experimental data from [17, 18] (a) and [19] (b) with theoretical profiles.

System (1)-(3) is supplemented by the thermal equation of state for an ideal gas. Within the framework of the turbulence model bieng used, the effective viscosity is a function of the TKE and the rate of its dissipation.

Allowing for the generation and disappearance of turbulence under the influence of body forces makes it necessary that we model the correlation $\langle \rho'u' \rangle$ on the basis of information on the turbulent flows of mass and heat. The flows of the substances being transported by turbulence are calculated from local values of three scalar quantities K, ε, and $\langle T'^2 \rangle$. The transport equations for these correlations are represented as follows:

$$\underbrace{\rho u \frac{\partial K}{\partial x} + \rho v \frac{\partial K}{\partial y}}_{\mathrm{I}} = \underbrace{\frac{\partial}{\partial y}\left(\frac{\mu_{ef}}{\sigma_K}\frac{\partial K}{\partial y}\right)}_{\mathrm{II}} + \underbrace{\mu_t\left(\frac{\partial u}{\partial y}\right)^2 - g\langle \rho'u' \rangle}_{\mathrm{III}} - \underbrace{\rho\varepsilon}_{\mathrm{IV}}; \tag{4}$$

$$\underbrace{\rho u \frac{\partial \varepsilon}{\partial x} + \rho v \frac{\partial \varepsilon}{\partial y}}_{\mathrm{I}} = \underbrace{\frac{\partial}{\partial y}\left(\frac{\mu_{ef}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial y}\right)}_{\mathrm{II}} + \underbrace{\frac{\varepsilon}{K}\left[C_{\varepsilon 1}\mu_t\left(\frac{\partial u}{\partial y}\right)^2 - C_{\varepsilon 3} g\langle \rho'u' \rangle\right]}_{\mathrm{III}} - \underbrace{C_{\varepsilon 2}\rho\frac{\varepsilon^2}{K}}_{\mathrm{IV}}; \tag{5}$$

$$\underbrace{\rho u \frac{\partial \langle T'^2 \rangle}{\partial x} + \rho v \frac{\partial \langle T'^2 \rangle}{\partial y}}_{\mathrm{I}} = \underbrace{\frac{\partial}{\partial y}\left(\frac{\mu_{ef}}{\sigma_{\langle T'^2 \rangle}}\frac{\partial \langle T'^2 \rangle}{\partial y}\right)}_{\mathrm{II}} + \underbrace{C_{T1}\mu_t\left(\frac{\partial T}{\partial y}\right)^2}_{\mathrm{III}} - \underbrace{C_{T2}\rho\frac{\varepsilon}{K}\langle T'^2 \rangle}_{\mathrm{IV}}, \tag{6}$$

where I is convection; II, turbulent diffusion; III, generation; and IV, viscous dissipation.

We used the values recommended in [13] for the constants in the model. The assumption that free convection causes anisotropy of the small-scale components of turbulence is the reason for the scatter of the values of C_{T2} used in the literature [8]. However, in the absence of direct empirical material on this problem, we decided to take the value used in [13] for C_{T2}.

We further postulate the existence of a connection between pressure and density pulsations

$$\langle \rho'u' \rangle = \frac{\langle T'\rho' \rangle}{\langle T'^2 \rangle}\langle T'u' \rangle,$$

where in the limiting case of small fluctuations

$$\frac{\langle T'\rho' \rangle}{\langle T'^2 \rangle} = \left(\frac{\partial \rho}{\partial T}\right)_{T=\langle T \rangle} = -\frac{\rho}{T}.$$

The additional assumptions have to do with the need to model the correlation $\langle T'u' \rangle$, which we write in the form [9]:

$$\langle T'u' \rangle = C_\rho (K\langle T'^2 \rangle)^{1/2}. \tag{7}$$

The turbulence model is closed by the equation for eddy viscosity

$$\mu_t = C_\mu q_k \rho \frac{K^2}{\varepsilon}, \tag{8}$$

where the coefficient

$$q_k = \frac{1}{1 + \left[\frac{\text{term III}}{\text{term IV}} - 1\right]} \tag{9}$$

in the limiting case of "equilibrium" turbulence (term III = term IV) is equal to 1. The difference between the coefficient and unity in other cases is connected with the lack of similitude between the transfer of shear stress and its generation.

The basic equations for the characteristics of pulsative motion are invalid near solid walls. It is therefore difficult to formulate boundary conditions, since they are generally a simplified form of laws of conservation. One method of correctly describing turbulent flow near solid walls introduces "boundary" functions [14] reflecting knowledge of the empirical characteristics of heat and mass transfer near walls. Using empirical data on local "equilibrium" turbulence in the wall region [3], we will formulate boundary conditions for K and ε. Here it follows from the "equilibrium" analog of the balance for K, without allowance for the generation term due to body forces, that

$$K_b = \tau_b / \sqrt{C_\mu}, \tag{10}$$

$$\varepsilon_b = \sqrt{C_\mu}\, K_b \left(\frac{\partial u}{\partial y}\right)_b. \tag{10a}$$

Equation (10a) is modified with allowance for the agreement between the Prandtl model proposed as a basis for describing boundary flow and the K–ε model at the boundary of the viscous sublayer [15]. $\tau_b = (\varkappa y_b)^2 \left(\frac{\partial u}{\partial y}\right)^2_{y=y_b}$, from which it follows that

$$\varepsilon_b = \frac{\tau_b^{3/2}}{\varkappa y_b}. \tag{11}$$

The value of τ_b was calculated using "boundary" functions modified to allow for free-convective flows. Also, the gradients of the quantities characterizing pulsative motion were assumed to have a value of zero near the wall.

The initial conditions for K, ε, and $\langle T'^2 \rangle$ are written as follows:

$$\begin{cases} K_0 = \text{const}\, u_0^2, & \text{const} \ll 1; \\ \varepsilon_0 = C_D \dfrac{K_0^{3/2}}{\text{const}\, L_0}, & \text{const} \leqslant 1; \\ \langle T_0'^2 \rangle = \text{const}\,(T_w - T_\infty)^2, & \text{const} \ll 1. \end{cases} \tag{12}$$

Thus, the formulation of the boundary conditions, which is subject to the solution of system (1)-(6), appears as follows:

$$\begin{cases} 0 < y < L_0 \text{ and } x = 0;\ u = u_0,\ T = T_0,\ K = K_0,\ \varepsilon = \varepsilon_0,\ \langle T'^2 \rangle = \langle T_0'^2 \rangle; \\ y = 0 \text{ and } x \geqslant 0;\ u = v = 0,\ T = T_w,\ K = K_w,\ \varepsilon = \varepsilon_w,\ \langle T'^2 \rangle = 0; \\ y \to \infty \text{ and } x \geqslant 0;\ T = T_\infty\ (= T_0);\ u = K = \varepsilon = \langle T'^2 \rangle = 0. \end{cases} \tag{13}$$

Let us examine an "equilibrium" analog of the turbulence model being used. Assuming that the generation of turbulence due to deformation of the mean flow and the effects of buoyancy are equal to its dissipation, Eqs. (4)-(6) degenerate into a system which describes the "local-equilibrium" case of turbulence:

$$\mu_t \left(\frac{\partial u}{\partial y}\right)^2 + C_\rho \frac{\rho}{T} g (K \langle T'^2 \rangle)^{1/2} = \rho\varepsilon; \quad C_{T1}\mu_t \left(\frac{\partial T}{\partial y}\right)^2 = C_{T2}\rho \frac{\varepsilon}{K} \langle T'^2 \rangle \tag{14}$$

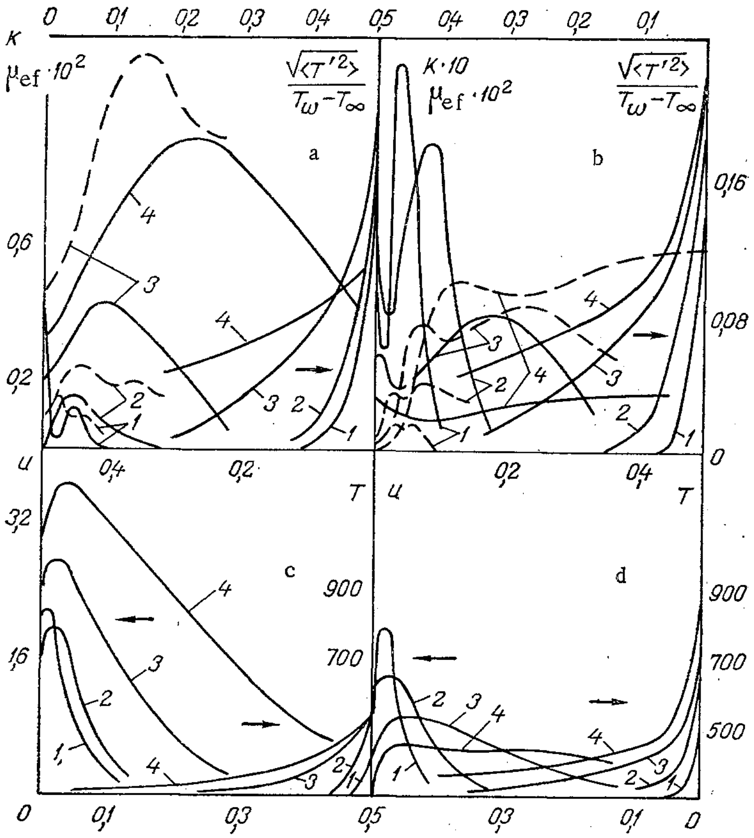

Fig. 2. Distributions of profiles of mean velocities, temperatures, TKE (solid line), and effective viscosity (dashed line) in a jet on a vertical heated plate: a, c) with allowance for buoyancy effects; b, d) without allowance for same; 1) x = 0.13 m; 2) 0.61; 3) 2.29; 4) 4.31.

Excluding $\langle T'^2 \rangle$ from the system of equations and considering the determination of the rate of dissipation of the TKE, we obtain

$$\mu_t\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Bg}{T}\mu_t\frac{\partial T}{\partial y} = C_D\rho\frac{K^{3/2}}{l_t}, \tag{15}$$

where l_t is the turbulence scale, while the constant $B = \sqrt{\frac{C_\rho^2 C_{T1}}{C_{T2}C_\mu}}$. Using Kolmogorov's well-known formula [16]

$$\mu_t = \rho C_\mu K^{1/2} l_t$$

and transforming Eq. (15), we obtain

$$\mu_{ef} = \mu_l + \frac{C_\mu^{3/2}}{C_D^{1/2}}\rho l_t^2\sqrt{\left(\frac{\partial u}{\partial y}\right)^2 + \frac{Bg}{T}\frac{\partial T}{\partial y}}. \tag{16}$$

Without allowance for the work of buoyancy (g = 0), Eq. (16) coincides with the Prandtl formula for effective viscosity.

The transport equations for pulsative correlations K, ε, and $\langle T'^2 \rangle$ are similar in structure to the main parabolic equations for mean flow. They can be solved simultaneously with the equations for mean flow by the Patankar–Spalding integrointerpolational method in the variables x, w ($w = \psi/\psi_\infty$ is a normalized stream function). A spatially nonuniform grid is used in the boundary region due to the considerable gradients of the individual variables, the dimensions of the body cells increasding from the wall to the periphery.

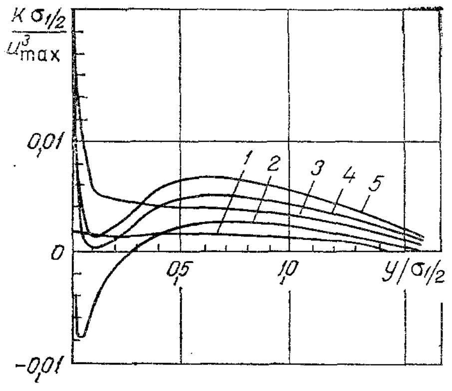

Fig. 3

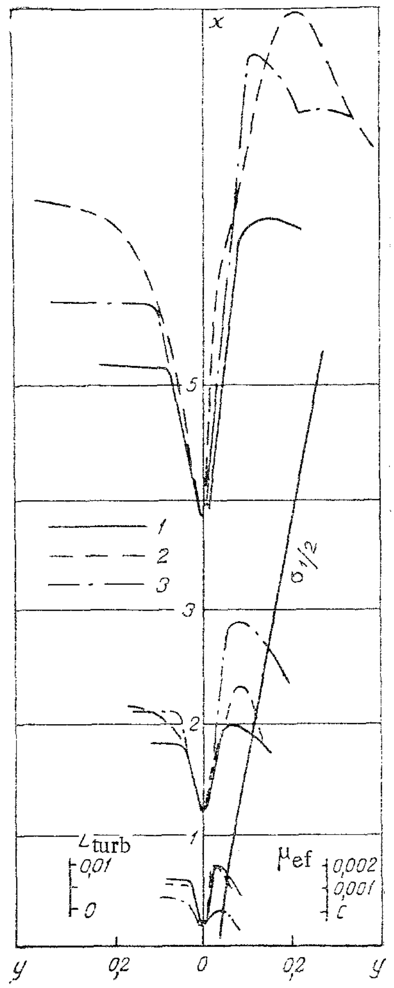

Fig. 4

Fig. 3. Distribution of individual terms in the TKE balance equation: 1) $\frac{g}{\rho} < \rho'u' >$; 2) $\frac{1}{\rho}(C - D)_k$; 3) ε; 4) $< u'v' > \frac{\partial u}{\partial y}$; 5) $\frac{g}{\rho} < \rho'u' > - < u'v' > \frac{\partial u}{\partial y}$.

Fig. 4. Diagram of distributions of effective viscosity and turbulence scale with $T_w = 1000°K$: 1) Prandtl formula; 2) $K-\varepsilon-< T'^2 >$ model; 3) modified Prandtl formula; x, y, $\sigma_{1/2}$, m; μ, kg/m·sec.

To compare calculated data with experimental results, we modeled the experiment conducted in [17, 18]. The subject of investigation was forced convection in a semiinfinite air jet traveling along a heated wall. It can be seen from Fig. 1 that the calculated distributions of u/u_0, $(T - T_\infty)/(T_w - T_\infty)$, and TKE agree with the experimental profiles. The difference in the empirical and numerical results in the peripheral region has to do with the inadequacy of the boundary-layer model in describing the actual flow near the free boundary of the jet. We took the data from [19] for a test experiment for the regime of free turbulent convection. By analogy with [19], we calculated the thickness of the layer δ_1, determined in accordance with a linear law of temperature change with respect to the transverse coordinate. We then compared experimental values of the ratio of the Archimedes number $Ar_1 = g(\rho_\infty - \rho_w)\delta_1^3/(\rho_\infty\nu^2)$ to the Reynolds number $Re_1 = u_{\delta_1}\delta_1/\nu$ with calculated values. The two sets of values agree in the region of developed turbulent convection.

Figure 2 presents diagrams showing the development of the profiles of velocity and temperature and the pulsative characteristics (TKE — solid line, effective viscosity — dashed line, dimensionless pulsations of temperature — solid line to the right) for different sections x in a jet developing along a heated wall. Figures 2a, c were plotted with allowance for buoyancy effects, while Figs. 2b, d were plotted without consideration of same. It is apparent that the presence of buoyancy qualitatively changes the structure of the flow. Whereas the maximum values of velocity and stress of the flow at the wall and the TKE increase as the flow develops if buoyancy is taken into account, these quantities decrease with increasing distance from the outlet section of the nozzle if buoyancy is ignored. The effective viscosity increases in both cases, although free convection increases the shear stress, while the region of maximum turbulent exchange approaches the wall. It is interesting to note that allowing for the free-convective source leads to thinning of the thermal boundary layer; the region occupied by the flow is heated more weakly, and the level of the temperature pulsations in the boundary flow decreases.

Figure 3 shows the behavior of the terms of the TKE balance equation in the section x = 3.85 m and at a wall temperature T_w = 500°K. Here the terms of the TKE balance equations are multiplied by the quantity $\sigma_{1/2}/u^3_{max}$. In the boundary region, where viscosity is important, generation of small-scale turbulence is accompanied by its viscous dissipation. At the point of the position of the maximum, the "deformed" part of the turbulence generation $\langle u'v' \rangle \partial\bar{u}/\partial y$ decreases to zero, and the turbulence energy is generated by buoyancy forces $g/\rho \langle \rho'u' \rangle$. In this region the curve showing the difference between convection and diffusion of the TKE $(1/\rho)(C - D)_k$ takes negative values. Large energy-containing eddies are generated as the free boundary is approached, the large size of these eddies leading to a long period of existence, with displacement in the direction of flow development ($(C - D)_k > 0$).

Analysis of the distributions corresponding to individual terms of the TKE balance equation show that the convection of the quantity $\langle T'^2 \rangle$ predominates over its turbulent diffusion at all points across the flow field.

The use of the "equilibrium" models of turbulence is shown in Fig. 4. Shown for comparison are the results of calculation of the characteristic turbulence scale and the effective viscosity using the Prandtl formula (solid line)

$$\mu_{ef} = \mu_l + \frac{C_\mu^{3/2}}{C_D^{1/2}} \rho l_t^2 \left(\frac{\partial u}{\partial y}\right), \tag{17}$$

the modified Prandtl formula (16) (dashed line), and the "two-parameter" formula (8). Based on analysis of the dimensions in the last case, we determined the turbulence scale from the algebraic relation

$$l_t = C_\mu \frac{K^{3/2}}{\varepsilon}. \tag{18}$$

Use is made in Eqs. (16), (17) of the Prandtl hypothesis of a "mixing length," which regards the turbulence scale as a "geometric" flow parameter.

The maximum deviation between the predictions of the "equilibrium" model and the K–ε–$\langle T'^2 \rangle$ model is seen on the initial section of jet development, as well as in the peripheral zone – where three-dimensional transport by large eddies is important. The "equilibrium" models of turbulence describe the distribution of the local linear scale in the internal part of the boundary layer well. In the region of predominance of free-convective flow (Fig. 4), modified formula (16), in comparison to Eq. (17), more reliably describes processes of turbulent exchange.

NOTATION

K, kinetic turbulence energy; u', velocity pulsation; ε, dissipation of kinetic turbulence energy; $\langle T'^2 \rangle$, mean square temperature pulsation; μ_l, laminar viscosity; L_0, nozzle dimension; u_0, initial velocity; T_w, wall temperature; ρ, density; u, v, components of velocity vector; g, acceleration due to gravity; μ_{ef}, effective viscosity; μ_t, eddy viscosity; T, temperature; σ, Prandtl number; σ_k, $C_{\varepsilon 1}$, $C_{\varepsilon 3}$, σ_ε, $C_{\varepsilon 2}$, $\sigma_{\langle T'^2 \rangle}$, C_{T1}, C_{T2}, C_ρ, C_μ, C_D, constants of the turbulence model; τ_w, friction stress on the wall; l_t, turbulence scale; ψ, stream function; $\sigma_{1/2}$, half-width of jet; Nu_x, Nusselt number; u_{max}, maximum velocity; (C – D), difference between convection and diffusion of individual transport quantity; b, subscript for first boundary component near the solid surface; ϰ, Karman constant.

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